

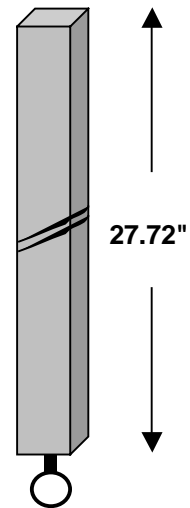
## Pascal's Principle - Paradox Lost

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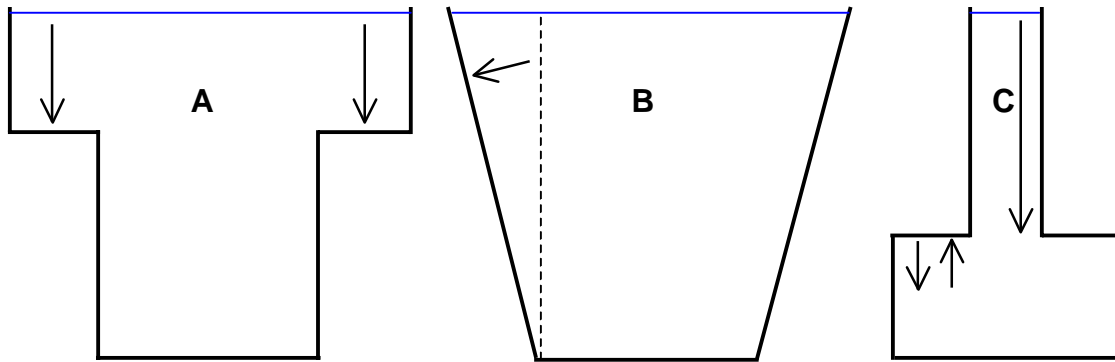
Have you ever wondered why the pressure, exerted by a column of liquid, has absolutely nothing to do with its volume or, for that matter, the geometric shape of its container? It certainly seems that volume, and the additional weight it can contribute, should be a factor. But, according to Blaise Pascal, pressure depends upon the density and height of the liquid and is completely independent of its volume and the shape of the container. Now this, "hydrostatic paradox", can be confusing so lets take a look at several examples.

Figure 1 shows a container that is 1" X 1" square and 27.72" (2.31') tall. Its volume of 27.72 cubic inches just happens to be the volume of one pound of water at standard temperature and pressure. If we mounted a pressure gauge at the base of the column, it would read 1 PSI - - just what we might expect from one pound of water resting on a base that has an area of one square inch. Increase the column size to 2" X 2" X 27.72" and the weight of the water quadruples to four pounds and, the gauge still reads 1 PSI. But, this relationship still makes sense because the area of the base has also quadrupled to four square inches and - - 4 lb / 4 sq in = 1 PSI. Let's go the other way and make it smaller - - say  $\frac{1}{2}$ " X  $\frac{1}{2}$ " X 27.72". Now the weight of the water is reduced to  $\frac{1}{4}$  pound but,  $\frac{1}{4}$  lb /  $\frac{1}{4}$  sq in still equals 1 PSI.



It appears that the changing area of the base may be the reason volume is not a consideration but, these examples can be a bit misleading because volume (and weight) just happens to be directly proportional to the area of the base. This, of course, is not always the case in the real world so lets take a look at several other containers and try to explain why volume and shape take a back seat to height and density.

In figure 2, containers **A**, **B**, & **C** are filled with liquid to the very same height and the pressure, measured at their bases, is exactly the same. The base dimensions of each are also the same but their shapes and volumes vary substantially. How can the pressure at their bases be the same?



Well, in the case of container **A**, could it be that the lower, horizontal surfaces of the expanded upper area support the weight of the additional liquid in that upper area? In fact, they do and that is why the pressure seen at the base is purely a function of the height of the liquid directly above it. But, what about container **B**? It has no horizontal surfaces available to support the, obviously, greater volume! And, how can that puny, contracted upper portion of container **C** influence the pressure seen at its base? Lets explore.

A brick, resting on the floor, will exert a downward force, in pounds per square inch, that is equal to its weight divided by its surface area ( $f = w/a$ ). Turn it on its side or end and PSI increases because surface area decreases. But, regardless of its position, it does not deform due to its weight. And, this ability to maintain its shape is characteristic of a solid. Unlike solids, liquids do deform and cannot maintain a geometric shape without the assistance of a container. But unlike the brick, a contained liquid does not produce just a downward force. Instead, as predicted by Pascal, it exerts pressure in every imaginable direction. And, these directional forces become particularly important when they are perpendicular to the walls of the container.

If you were to draw two vertical lines, from the edges of the base of container **B**, upwards to the surface of the water you would end up with a right triangle on either side. It turns out that the weight of the additional water contained within these triangles is supported by the angled sides of the container. The reason they

are able to provide this support is because the net force of the water in the triangle is directed perpendicularly towards them. Therefore the force or pressure on the base of the container is due solely to the height of the water directly above it.

Container **C**, however, is very different than the other two. Its upper portion is contracted and is missing two thirds of the volume it would contain if its width were the same as the lower portion. How can it possibly exert the same pressure over the much larger area of the container's base? Remember that a liquid exerts its pressure in all directions and the liquid in the lower portion will therefore exert an upward pressure against the horizontal surfaces above. The amount of pressure exerted depends upon the height of the upper portion above that horizontal surface. Newton's third law tells us that the horizontal surface will exert an equal pressure in a downward direction. And, it is that downward pressure that makes up for the missing liquid in the upper portion. Once again, the pressure seen at the base of the container is due solely to the height of the liquid above it.

Regardless of the shape or volume of the container (tank, reservoir, whatever), the pressure exerted by any liquid will always be proportional to its height and density. Even complex shapes such as a sphere or horizontal cylinder adhere to the same rule. For water (density = 1) each 2.31' of height will result in 1 PSI of pressure. Less dense liquids such as kerosene (density = 0.8) will produce a lower pressure (0.8 PSI) for each 2.31'. And heavier liquids will produce proportionally higher pressures.

One final word about pressure - - when the pump industry refers to pressure we are usually talking about "gauge pressure" or PSIG. Standard pressure gauges are calibrated to read zero at sea level and thus ignore atmospheric pressure. "Absolute pressure" or PSIA does not ignore atmospheric pressure and gauges calibrated in this manner will display 14.7 PSI at sea level.

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