

Inertia - Confusing Units Revisited

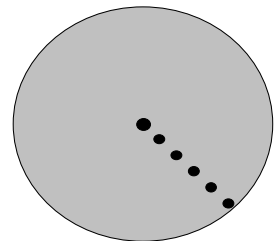
[Joe Evans, Ph.D](#)

<http://www.PumpEd101.com>

In the October 2007 issue of P&S, we took a look at how several units of measure differ in a linear versus a rotational environment. For example, that simple linear unit called force (f) becomes far more complex in a rotating system. Its rotational counterpart, torque, not only consists of force but also the radius and angle at which that force is applied. This increased complexity also applies to another important unit - - inertia. Why is inertia important? There are times when we need to calculate the amount of torque that will be required to start or stop a rotating machine.

In a linear frame of reference inertia is pretty simple. As defined by Newton's first law of motion, inertia is the tendency of a body in motion to remain in motion in a straight line and at a constant velocity unless acted upon by some outside force. This same law holds true for an object that is at rest. Since linear motion occurs in a straight line, velocity is simply distance / time (d/t) and is often measured in terms of miles per hour or feet per second. The inertia of an object moving in a straight line is also proportional to its momentum (mv) so, a change in either its mass (weight) or velocity results in a similar change in its inertia. The reason this relationship holds true is because every part of that object moves at the same velocity. For example, if the front bumper of a truck is moving at 50 mph, so does its back bumper and every portion in between. Because of this uniform velocity, it makes no difference how the truck's weight is distributed. It could be concentrated in any location and its inertia will still be proportional to mv .

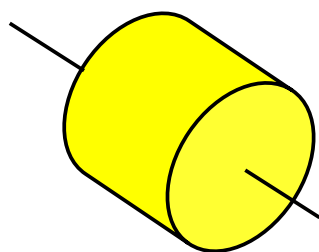
Things are quite different in the case of rotational motion. In fact none of the statements above apply! Take, for example, the simple disc seen in Figure 1. During rotation a point on its rim moves at a greater velocity than other points closer to its center and, at its exact center there is no motion at all. For this reason we normally do not measure rotational velocity in linear units like feet per second unless we specify some exact radius. A more useful term is one that describes the number of complete rotations during a unit of time - - rotations per minute (RPM) for example. Even though there are an infinite number of points along its radius and each of them



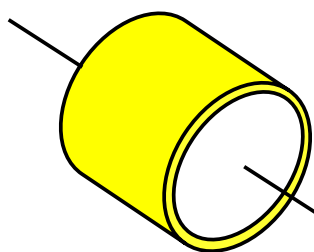
move at different velocities, all complete a single rotation at the very same time. If, however, we want to calculate rotational momentum it is important that we know the actual velocities of those points.

The angular velocity of any point on a rotating disc is $\omega = \Delta\theta/\Delta t$ (where $\Delta\theta$ is the change in the angle and Δt is the change in time). If we consider $\Delta\theta$ to be one complete rotation (360 deg), the distance that a point on that disc will travel is its circumference or $2\pi r$ (where r is the radius from the center of rotation to that point). If we let Δt equal one minute, we can then use the equation $v = 2\pi r\omega$ (where v is the equivalent linear velocity and ω is rpm) to compute the velocity of any point on the radius and express it in linear units such as feet per minute. As expected rotational velocity is a bit more complicated than the simple $v = d/t$ definition of linear velocity.

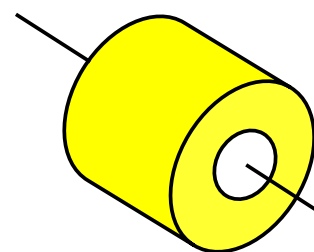
It is even more complex with respect to mass. If there are an infinite number of points on the radius of a disc, then there must be an infinite number of circular masses. And, unlike our truck example, their location is important as they will have a substantial influence on the inertia of a stationary or rotating disc. Again the inertia of a rotating disc is proportional to its momentum but, it is not as simple as our linear example (mv). Instead, the momentum of each bit of circular mass is equal to mvr where r is a particular bit's distance from its axis of rotation. If this is the case then the disc's total momentum is equal to the sum of all its individual momentums. Calculating the total momentum or inertia of an infinite number of circular masses that travel at different velocities could be a formidable mathematical task. Fortunately physics, with the help of calculus, has derived a number of simple equations that allow calculation of the rotational inertia of various geometric shapes. In the case of a complex shape, it can often be broken down into several simple shapes that can employ these same equations. Figure 2 shows three different cylinders with a rotational axis illustrated by the straight black line.



$$I = \frac{1}{2}MR^2$$



$$I = MR^2$$

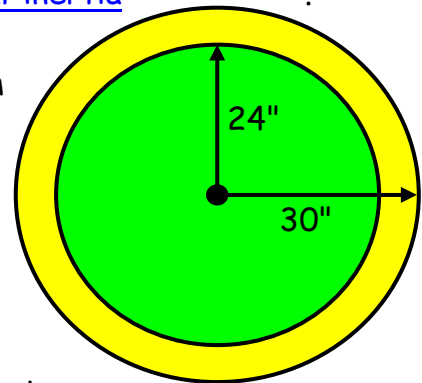


$$I = \frac{1}{2}M(R_1^2 + R_2^2)$$

The one on the left is completely solid while the one on the right has a hollow center. The one in the middle takes the form of an extremely thin shell. The equations show the moment of inertia (I) for each configuration. Note that " I " depends upon the mass and radius of the cylinder and where that mass is concentrated. It has nothing to do with its length. (Length, however, would become a component and replace R if the cylinder was rotating about a perpendicular axis through its center.) If the radius and mass were the same for all three, the shell like cylinder would possess the most inertia and the solid cylinder would have just half that amount. The one with the hollow center would fall somewhere in between based upon the values of R_1 and R_2 (R_1 is the distance between the axis and the inner radius of the cylinder while R_2 is the distance between the axis and the outer radius). If you would like to see how these equations (and others) are derived go to

<http://hypertextbook.com/physics/mechanics/rotational-inertia>

Figure 3 is an example of a cast iron flywheel made from a single casting. The green section is 2" thick and has a radius of 24". The yellow section is 5" thick and extends 6" beyond the perimeter of the green section. How can we determine the moment of inertia for this somewhat more complex shape?



It turns out that our flywheel is a combination of two of the shapes we just discussed - - a solid cylinder (green) and a hollow cylinder (yellow). If we determine the weight (mass) of each we can use those two simple equations to determine the inertia of each. The sum of these two inertias is the total inertia of the flywheel.

You may have noticed that I defined "RPM" as rotations per minute not revolutions per minute. I have received several emails questioning this in the past so I will answer it here. Although rotation and revolution are often used interchangeably, they are quite different. When an object revolves it moves about another object. For example, the earth revolves about the sun. When an object rotates it moves about itself so the earth rotates about its axis once every twenty four hours. Therefore when we use the term RPM to describe the speed of an electric motor or centrifugal pump R stands for rotations.

Joe Evans is responsible for customer and employee education at PumpTech Inc, a pumps & packaged systems manufacturer & distributor with branches throughout the Pacific Northwest. He can be reached via his website www.PumpEd101.com. If there are topics that you would like to see discussed in future columns, drop him an email.