# Confusing Units of Measure - Work, Power, \& Torque 

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Back in the early seventies, when I was in grad school, our government pledged to migrate the US into the metric system of measure. A popular cartoon, at the time, showed a lab technician with a box of amputated human feet standing at the door of the supply room. The supply clerk was also holding a box but, his was full of volt meters. The caption - - "Trading Feet for Meters". That was almost thirty-seven years ago and we still have most of those feet! I guess that I could say that we are still "inching" into the metric system.

The SI system (Système international d'unités) is the modern day version of the metric system and the US gets a lot of grief for not embracing its inherent transportability across international boundaries. It is the standard in our scientific community and its use is increasing in our engineering and industrial sectors but, it is still pretty unpopular with the proletariat. Why ? Because the units of measure are totally unusable by most of us. Take, for example, that simple quantity we know as work.

Work, in a linear environment, is pretty straight forward (pun intended). It is defined as the product of the force that is applied to an object and the distance that object travels as a result of that force. So work is simply w=fd. In the good ole English system force is measured in pounds and distance is measured in feet. So, if we lift a 100 pound box to a height of 10 feet we perform 1000 foot-pounds (ft-lb) of work. Since work is directly proportional to both force and distance, a 10 pound box lifted to a height of 100 feet requires exactly the same amount of work.

Now, if we moved this box outside the US, SI would take over. First we would convert pounds to kilograms and feet to meters. Then work in SI units is the joule and the joule is defined as a newton-meter. And, we all know the a newton (named after Sir Isaac) is the amount of force required to accelerate a mass of one kilogram at a rate of one meter per second squared. Yea right! It could be worse. Before SI, a unit of work could also be the erg which is a dyne-centimeter. And a dyne is a gramcentimeter per second squared. Figure 1 shows how yet another cartoon of the period put the relationship of the erg and dyne-centimeter into terms we could understand.

erg (dyne-centimeter)

The English system equation for work tells us exactly how much work is performed and it does so in units that are quite understandable. In fact gallons are easily converted to pounds and we can use that same simple equation to evaluate the work done by a pumping system. But, what it does not tell us is how quickly that work gets done. When we lift that 100 lb box to a height of 10 ft we perform $1000 \mathrm{ft}-\mathrm{lb}$ of work. It doesn't matter
if it takes 10 seconds or 10 days, it is still 1000 ft -lb of work. The rate at which work is done is referred to as power and power is equal to the work performed divided by the time it takes to perform it. In simple terms, power = w/t. If it takes one minute to lift that 100 lb box to a height of 10 ft the power required, in English units, is $1000 \mathrm{ft}-\mathrm{lb} / \mathrm{min}$. Again, pretty straightforward. Now, you might think that the SI system, where power is measured in watts, would be equally understandable. But, unfortunately, it isn't because the watt is defined as a joule per second. And that brings us back to the newton-meter.

Thanks to a fellow named James Watt, there is a more meaningful way to relate the watt to the ft-lb. In the late 18th century he made some major improvements to the steam engine - - improvements that made it a viable alternative to other sources of power. One of the power hungry applications at the time was coal mining and most were powered by horses. What Watt needed was a way to compare the power of his engine to that of a team of horses. It is said that, through experimentation, he determined that the average horse could lift about 182 pounds to a height of 181 feet in one minute (there are several versions of this story but the end result is the same). Thus the power, or horsepower in this particular case, is $33000 \mathrm{ft}-\mathrm{lb} / \mathrm{min}$. It turns out that this quantity is equal to 745.7 joules/sec in the SI system. One joule per second was named a watt in his honor so one HP is equal to approximately 746 watts. In the US we still rate an electric motor's power output in HP, while most other countries use kilowatts (kW).

Now, it is not difficult to visualize the work done in Watt's experiment. Just drop a long rope down a mine shaft, place it over a pulley, and hook the other end to a horse. The distance the horse walks times the weight he lifts is the work done. Measure how far he goes in one minute and you get horsepower. But, work and power can be a bit harder to visualize in a rotational environment. Take an electric motor for example - - how the heck do you measure force and distance?

In order to make some sense out of this we have to introduce a new term. If Watt's horse were to hold that weight in a static position no work would be done but the force holding it there would be similar to something we call torque. Torque can be considered the rotational counterpart of force and is defined as the product of force and the lever arm. Its SI unit is the newton-meter and its English equivalent is the foot-pound ( $\mathrm{ft}-\mathrm{lb}$ ). But wait, isn't that the unit for work? Yes it is, so torque is often called a pound-foot (lbft ) or a foot-pound-force (ft-lbf) in order to differentiate it from work. Unfortunately, this definition is not always self evident.

The difference between torque and linear force is the influence provided by the lever arm. Linear force acts in the same direction as the object it moves. The force created by torque acts at an angle and is therefore perpendicular to the movement it causes. Because of this, it consists of two components - - the force (F) applied in pounds and its point of application (r), measured in feet, from the center or axis of rotation ( $\mathbf{t}=\mathrm{Fr}$ ). Figure 2 shows how these

two components work together to produce torque. The amount of rotational force that is applied to a nut by a wrench depends upon the amount of force (F) applied to the handle and where that force is applied ( $r$ ). The torque that is generated is directly proportional to both. If 10 pounds is applied one foot from the nut, the result is $10 \mathrm{lb} / \mathrm{ft}$ of torque. If you move that force in by $1 / 2$ foot, torque is reduced to $5 \mathrm{lb} / \mathrm{ft}$. But, if you increase the force to 20 pounds, the torque at $1 / 2$ foot returns to $10 \mathrm{lb} / \mathrm{ft}$.

So far so good but, there is another factor that can affect the torque generated by that simple wrench. Suppose we applied that force on the handle at a 45 degree angle rather than at 90 degrees as shown in Figure 2. In this case some of the force would be directed downward, just as before, but some would also be directed toward the nut. How do we calculate the torque produced? Fortunately some simple trig can expand our equation for torque and make it useable regardless of the angle of the force. Its new form becomes $t=F(r \sin \theta)$ where $\theta$ is the angle. The sine of 90 degrees is 1 so when the force is perpendicular torque is simply Fr. The sine of 45 degrees is 0.707 and, at this particular angel torque becomes $F(r \times 0.707)$.

A good example of the effect that the angular and radial components of force can have on torque is the piston engine. When the spark plug fires, the piston is at the top of the cylinder and the crankshaft is at 0 degrees. As the piston moves down the cylinder the crankshaft rotates, torque is generated, and reaches its maximum at 90 degrees (half way through its down stroke). During the second half of the down stroke, torque decreases and disappears at 180 degrees. If you would like to see an excellent animation that illustrates this, go to http://science.howstuffworks.com/fpte4.htm .

In a linear system we saw that Work = Fd. In a rotational system torque ( $t$ ), which consists of force and radius, replaces force (F) and distance (d) becomes the angle through which the torque acts. Therefore Work $=\mathrm{t} \theta$ where $\theta$ is the angle. In the case of an electric motor, $\theta$ is 360 degrees or one complete rotation. It should follow that the power or horsepower produced by a rotational system is simply work divided by time. Unfortunately it is not that simple because time and rotational distance are related by angular speed. This adds a bit of mud to the equation and power becomes the product of torque and angular speed and angular speed is measured in radians per unit of time!

Fortunately we can convert angular speed to a more familiar unit -- RPM. Now our equation becomes $P=t \times 2 \pi \times R P M$ and power is in ft/lb per minute. Why $2 \pi ?$ Because there are $2 \pi$ radians in one complete rotation. If you divide the result by the work done by Watt's horse in one minute you end up with horsepower. But there is an even easier way to calculate HP. Simply stated, HP = (t x rpm) / 5250. So where did that convenient conversion factor of 5250 come from? Well, we just divided the work done by Watt's horse (33000) by $2 \pi$ and got rid of both of them! If we know the horsepower we can rearrange that equation to $t=(H P \times 5250) / r p m ~ a n d ~ c a l c u l a t e$ torque.

As you can see from the equation in the previous paragraph, if we keep HP constant torque will vary inversely with rotational speed. For example, a 100 HP electric motor rated at 3500 rpm produces approximately $150 \mathrm{lb}-\mathrm{ft}$ of torque. If it were rated at 1750 rpm its torque would increase to about $300 \mathrm{lb}-\mathrm{ft}$. This is exactly what we would expect since HP or power is the rate at which work is done. If a 1750 rpm motor is to
accomplish the same amount of work in the same amount of time as its 3500 rpm cousin, it must do twice the work per rotation.

Figure 3 shows the torque and current curves of a typical, Class B, AC motor. The $y$ axis shows the \% of full load torque and current while the x axis shows the \% slip and the synchronous speed (Ns). Synchronous speed is the speed at which the magnetic field rotates about the stator and it is equal to ( 120 xhz ) / \#poles. At 60 hz a two pole motor has a synchronous speed of 3600 RPM. For a four pole
 motor that speed is reduced to 1800 RPM. Slip or slip speed is the actual rotational speed of the rotor and it is typically 3 to $5 \%$ lower. The reason that an induction motor operates below synchronous speed is pretty simple. If the rotor were to move at the same speed as the rotating magnetic field in the stator there would be no relative motion between them. Therefore no flux lines would cut across the rotor bars and no magnetic field would be induced.

The torque produced by an induction motor is proportional to the strength of the interacting magnetic fields in the stator and rotor and varies, substantially, from zero to rated slip speed. The locked rotor or starting torque is the minimum torque that is developed when the rotor is at rest ( $\mathrm{RPM}=0$ ). The current required to produce this starting torque ranges from five to seven times the actual full load current. Pull up torque is the minimum torque developed by the motor as it accelerates from rest to the speed at which maximum torque occurs. As you can see, current drops very little during this period of acceleration. As the rotor approaches about $75 \%$ of its rated slip speed, torque reaches its maximum (breakdown torque) and current begins to drop. As the rotor continues to accelerate torque and current drop quickly and reach their full load values at the rated slip speed of the motor.

Figure 3 also shows us what happens when a motor is over loaded. Take, for example, that 100 HP / 1750 RPM motor we spoke about earlier. As the load is increased, torque increases and will reach its maximum at about $25 \%$ slip. If you substitute the new values of torque and speed into the HP equation you will find that HP increases from 100 to about 230! If the load continues to increase, torque will begin to decrease and rotation will come to a grinding halt. Chances are that you would not see this happen due to the large volume of smoke exiting the motor housing!

Hopefully this has made work, torque, and power in a rotational environment a little more understandable. I have also had a little fun trashing the metric system even though I believe we should have adopted it a long time ago. But those who expected it to be an easy transition must have been wearing blinders. Can you imagine going to

McDonalds for a 0.11339 kilogramer (maybe even with cheese)? Or, how about first down and 9.144 on the 45.72 (that would be mid field !). If we are going to make this system the standard, it will have to start in grade school.

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